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CHAPTER

PERMUTATION AND COMBINATIONS

QUICK REVISION OF THE CHAPTER

IMPORTANT CONCEPTS

Permutation:

Understanding the meaning of 'n' factorial:

Taking any number as 'n'. The Factorial of such number would be 'n' factorial and is denoted as $n!$ or n

This means product of all integers from n upto 1. So $n! = n (n - 1) (n - 2) \dots 3, 2, 1$. It can be mentioned that factorials are possible only for positive integers.

Permutation of n different objects taking r at a time ($r \leq n$)

The word permutation means Arrangement. Total number of permutations or arrangements that can be made out of 'n' different objects taking 'r' at a time (when $r \leq n$) is given as nP_r .

Classification of Permutations:

- (1) **When repetition is not allowed:** With every object being selected, the availability of the remaining objects do decrease with the selection of objects from the availability. Basically, the total number of objects available for selection decreases after every selection is done.
- (2) **When Repetition is allowed:** When the objects to be selected can have repetition without any restriction. In simple words, 'n' different objects could be taken 'r' at a time.

Formula: n^r

- (3) **Permutation of cases where distinction of objects is not possible:** When objects are similar in nature and cannot be distinguished among themselves or the identical distinction of any object from the available objects is not possible due to objects being identical, in such cases if permutation would be done considering all a like objects to be one single object. Thus, while finding number of possible permutations, the denominator would be divided by the number of permutation of a like objects as well.

The permutation of 'n' objects of which 'p' are alike of one kind and 'q' are alike of another kind is $n!/(p! q!)$

(4) Circular Permutation: When the objects are arranged in a circular formation then that is called Circular Permutation.

- (i) No. of circular permutations of 'n' different objects taking all at a time is $(n - 1)!$
- (ii) No. of circular permutations of 'n' different taking all at a time is $(n - 1)!/2$ when clockwise or anti clockwise arrangements are not different.

Combination:

Combination means selection or choice or formation of a group.

Each of different groups of selection that can be made by taking some or all of the no. of things at a time is called combination.

Combination of 'n' different objects taking 'r' at a time (when $r \leq n$). It is denoted by nC_r .

Important Formulas

- (1) $nP_r = n!/(n - r)!$
- (2) $nC_r = n!/r! (n - r)!$
- (3) $nP_r = r! nC_r$
- (4) $nC_r = nC_{n-r}$
- (5) $nC_n = 1$
- (6) $nC_0 = 1$
- (7) $nC_1 = n$
- (8) $nC_r + nC_{r-1} = (n + 1)C_r$
- (9) Total No. of combinations of n different things taken one or more things at a time is $2^n - 1$.

PAST EXAMINATION QUESTIONS

THEORY QUESTIONS

Q. 1. The value of $0!$ is:

- (a) 0
- (b) 1
- (c) Infinity
- (d) None of these

[Dec. 2016] [Modified]

Ans. (b) 1

Q. 2. A polygon of n sides has two diagonal. The value of n is

- (a) 4
- (b) 3
- (c) 8
- (d) 10

[June 2024]

Ans. (a) 4

Note: For a polygon two have diagonal, number of sides should be 4.

NUMERICAL QUESTIONS

Q. 1. The ratio of $({}^{2n}P_n + {}^{2n}C_n)$: $({}^{2n}P_n - {}^{2n}C_n)$ is:

(a) $(n + 1): (n - 1)$

(b) $(n - 1): (n + 1)$

(c) $(n! + 1): (n! - 1)$

(d) $(n + 1)!: (n - 1)! \quad [June\ 2013]$

Ans. (c) $(n! + 1): (n! - 1)$

Working Note:

$$\begin{aligned} &({}^{2n}P_n + {}^{2n}C_n): ({}^{2n}P_n - {}^{2n}C_n) \\ &= ({}^{2n}C_n \times n! + {}^{2n}C_n): ({}^{2n}C_n \times n! - {}^{2n}C_n) \\ &= {}^{2n}C_n (n! + 1): {}^{2n}C_n (n! - 1) \\ &= (n! + 1): (n! - 1) \end{aligned}$$

Q. 2. How many different odd numbers of 4 digits that can be formed with the digits 1, 2, 3, 4, 5, 6, 7; the digits in any number being all different?

(a) 480

(b) 240

(c) 620

(d) 440 [Sept. 2014]

Ans. (a) 480

Working Note:

For the number to be odd, the digit at unit place should be 1, 3, 5 or 7 and rest 3 digits can be any number from the rest, Therefore,

$$\begin{aligned} &4P_1 \times 6P_3 \\ &= (4!/3!) \times 6!/3! \\ &= (4 \times 3!/3!) \times (6 \times 5 \times 4 \times 3!/3!) \\ &= 4 \times 120 \\ &= 480 \text{ ways} \end{aligned}$$

Q. 3. A person has got 15 acquaintances of whom 10 are relatives. In how many ways may be invite 9 guests so that 7 of them would be relatives?

(a) 300

(b) 600

(c) 900

(d) 1,200 [Sept. 2014]

Ans. (d) 1,200

Working Note:

Out of 15 acquaintances 10 are relative, 7 relatives are to be invited out of total 9. Therefore,

$$\begin{aligned} \text{No. of ways} &= {}^{10}C_7 \times {}^5C_2 \\ &= (10!/7!3!) \times (5!/3!2!) \\ &= ((10 \times 9 \times 8)/6) \times ((5 \times 4)/2) \\ &= 120 \times 10 \end{aligned}$$

= 1,200 Ways.

Q. 4. Simplify: ${}^4P_2 \div {}^4C_2$

(a) 4

(b) 2

(c) 6

(d) 8 [Sept. 2014]

Ans. (b) 2

Working Note:

$$\begin{aligned} &{}^4P_2 \div {}^4C_2 \\ &= (4!/2!)/(4!/2!2!) \\ &= (24/2)/(24/4) \\ &= 12/6 \\ &= 2 \end{aligned}$$

Q. 5. ${}^{13}C_9 - {}^{12}C_8$ is equal to:

(a) ${}^{12}C_9$

(b) ${}^{12}C_8$

(c) ${}^{12}C_7$

(d) ${}^{13}C_7$ [Dec. 2014]

Ans. (a) ${}^{12}C_9$

Working Note:

$$\begin{aligned}
 & {}^{13}C_9 - {}^{12}C_8 \\
 &= (13!/9! 4!) - (12!/8! 4!) \\
 &= \{13! - (12! \times 9)\}/(9! 4!) \\
 &= \{12! (13 - 9)\}/(9! 4!) \\
 &= 12! \times 4/(9! 4 \times 3!) \\
 &= 12!/9! 3! \\
 &= {}^{12}C_9
 \end{aligned}$$

Q. 6. There are 20 stations on a railway line. How many different kinds of single first-class tickets must be printed, so as to enable a passenger to go from one station to another?

- (a) 380
(b) 360
(c) 480
(d) 350

[Dec. 2014]

Ans. (a) 380

Working Note:

Single first-class tickets will connect any two stations one way so total tickets to be printed $20C_2$. Also, each ticket will be printed both ways i.e. both to and fro from between any two stations.

$$\begin{aligned}
 \text{Therefore, Total Tickets} &= 2 \times 20C_2 \\
 &= 2 \times (20 \times 19)/(2 \times 1) \\
 &= 380 \text{ tickets}
 \end{aligned}$$

Q. 7. Find the value of n , If ${}^nP_4 : (n - 1)P_3 = 10$.

- (a) 8
(b) 6
(c) 10
(d) 12

[Dec. 2014]

Ans. (c) 10

Working Note:

$$\begin{aligned}
 {}^nP_4 : (n - 1)P_3 &= 10 \\
 n!/(n - 4)! : (n - 1)!/(n - 4)! &= 10
 \end{aligned}$$

$$n(n - 1)! : (n - 1)! = 10$$

$$n = 10$$

Q. 8. The number of permutations if the letters in the word "BANANA" in which two letters N do not come together is:

- (a) 40
(b) 60
(c) 80

(d) 100 [March 2015, June 2015, Dec. 2015, June 2016]

Ans. (a) 40

Working Note:

$$\text{Total Possible Permutations} = 6!/(2!3!) = 720/(2 \times 6) = 60$$

Permutations where both N come

$$\text{together} = (5! 2!)/(2!3!) = (120 \times 2)/(2 \times 6) = 20$$

Therefore, number of combinations where two letter N do not come together = $60 - 20 = 40$

Q. 9. ${}^nP_r = 720$ nC_r , then r :

- (a) 4
(b) 5
(c) 6
(d) 8

[March 2015]

Ans. (c) 6

Working Note:

We Know that ${}^nP_r = {}^nC_r \times r!$, Therefore,

$${}^nP_r = 720 {}^nC_r$$

$${}^nC_r \times r! = 720 {}^nC_r$$

$$r! = 720$$

$$r! = 6!$$

$$r = 6$$

Q. 10. The total number of arrangements in which 5 positive signs and 3 negative signs can be placed in a row such that no two negative signs never come side by side is:

(a) 15

(b) 20

(c) 720

(d) None of these [March 2015]

Ans. (b) 20

Working Note:

Total 8 places are there two arrange the 8 signs. If no two negative signs are to come together than they can be placed on three out of any five places (*i.e.* two ends and three places in between positive signs) and remaining 5 positive signs on remaining 5 places. This can be done two ways (*i.e.* Negative sign occupying odd place or even place). Therefore, total arrangements =

$$(5P_3 \times 5P_5 \times 2)/5!3!$$

$$= (5!/2!) \times 5! \times 2/5!3!$$

$$= (120/2) \times 2/3!$$

$$= 120/6$$

$$= 20 \text{ Ways}$$

Q. 11. If $mC_6 : (m - 3)C_3 = 91:4$, then the value of m is:

(a) 13

(b) 15

(c) 14

(d) None of these [March 2015]

Ans. (b) 15

Working Note:

$$mC_6 : (m - 3)C_3 = 91:4$$

$$m!/6! : (m - 3)!/3! = 91:4$$

$$m(m - 1)(m - 2)(m - 3)!/720 : (m - 3)!/6 = 91:4$$

$$m(m - 1)(m - 2)/120 = 91/4$$

$$m(m - 1)(m - 2) = 2730$$

$$m(m - 1)(m - 2) = 15 \times 14 \times 13$$

$$m = 15$$

Q. 12. A man has 6 friends. The total number of ways so that he can invite one or more of his friends is equal to:

(a) 64

(b) 60

(c) 720

(d) 63 [June 2015, June 2017]
[Modified]

Ans. (d) 63

Working Notes:

Total number of ways in which one or more can be invited is $= 2^n - 1$

$$= 2^6 - 1$$

$$= 64 - 1$$

$$= 63 \text{ Ways}$$

Q. 13. How many numbers can be formed between 100 to 1000 out of 1, 3, 4, 7, 8 without repetition of any number.

(a) 60

(b) 84

(c) 120

(d) 93 [Sept. 2015]

Ans. (a) 60

Working Note:

Numbers Between 100 and 1000 can be only 3 digit numbers. So total numbers without repetition are $= 5P_3$

$$= 5!/2!$$

$$= 120/2 = 60$$

Q. 14. If ${}^{11}P_r = 110$, then the value of r is:

(a) 2

(b) 10

(c) 4

(d) None of these [Sept. 2015]

Ans. (a) 2

Working Note:

$${}^{11}P_r = 110$$

$$11!/(11-r)! = 110$$

$$11 \times 10 \times 9!/(11-r)! = 110$$

$$9! = (11-r)!$$

$$9 = 11-r$$

$$r = 2$$

Q. 15. 5 letters can be posted in 4 letter boxes in:

(a) 256 ways

(b) 1024 ways

(c) 625 ways

(d) None of these

[Sept. 2015 March 2016]

Ans. (b) 1024 ways

Working Notes:

Each letter can be posted in any of the four letter boxes, so the number of ways =

$$4P_1 \times 4P_1 \times 4P_1 \times 4P_1 \times 4P_1$$

$$= 4 \times 4 \times 4 \times 4 \times 4$$

$$= 1024 \text{ Ways}$$

Q. 16. If ${}^n P_3 = 120$ then n :

(a) 8

(b) 4

(c) 6

(d) None of these

[Dec. 2015, June 2016]

Ans. (c) 6

Working Note:

$${}^n P_3 = 120$$

$$n!/(n-3)! = 120$$

$$(n)(n-1)(n-2)(n-3)!/(n-3)! = 120$$

$$(n)(n-1)(n-2) = 120$$

$$(n)(n-1)(n-2) = 6 \times 5 \times 4$$

$$\text{So, } n = 6$$

Q. 17. The value of ${}^{12}P_2 + {}^8P_3$ is:

(a) 648

(b) 468

(c) 846

(d) None of these [March 2016]

Ans. (b) 468

Working Note:

$${}^{12}P_2 + {}^8P_3$$

$$= 12!/10! + 8!/5!$$

$$= (12 \times 11) + (8 \times 7 \times 6)$$

$$= 132 + 336$$

$$= 468$$

Q. 18. The total numbers of arrangements of the letters in the expression $X^3 Y^2 Z^4$ when written in full length is:

(a) 2520

(b) 1260

(c) 610

(d) None of these [March 2016]

Ans. (b) 1260

Working Note:

There are total 9 terms out of which x is repeated thrice, y twice and z four times, so total arrangements = $9!/3!2!4!$

$$= 3,62,880/(6 \times 2 \times 24)$$

$$= 3,62,880/288$$

$$= 1260$$

Q. 19. There are 10 lamps in a room. Each one of them can be switched on independently. The number of ways in which the hall can be illuminate is:

(a) 100

(b) 1024

(c) 1023

(d) 10

[June 2016]

Ans. (c) 1023

Working Note:

Hall can be illuminated with one or more lamps. So, total ways = $2^n - 1$

$$= 2^{10} - 1$$

$$= 1024 - 1$$

$$= 1023 \text{ ways}$$

Q. 20. How many numbers can be formed between 100 to 1000 out of 1, 3, 4, 7, 8 if repetition of any number is allowed?

$$(a) 60$$

$$(b) 84$$

$$(c) 125$$

$$(d) 92$$

[June 2016]

Ans. (c) 125

Working Note:

Numbers Between 100 and 1000 can be only 3 digit numbers. So total numbers with repetition are:

$$5P_1 \times 5P_1 \times 5P_1$$

$$= 5 \times 5 \times 5 = 125$$

Q. 21. If ${}^5C_r - {}^3C_2 = {}^7C_1$, then r is:

$$(a) 4$$

$$(b) 3$$

$$(c) 2$$

$$(d) \text{ both 2 and 3}$$

[June 2017]

Ans. (d) both 2 and 3

Working Note:

$${}^5C_r - {}^3C_2 = {}^7C_1$$

$${}^5C_r - 3 = 7$$

$${}^5C_r = 10$$

Now, 5C_2 and 5C_3 both gives the value of 10, Therefore,

$${}^5C_r = {}^5C_2$$

$$r = 2$$

or

$${}^5C_r = {}^5C_3$$

$$r = 3$$

Q. 22. If ${}^{16}p_r = 240$, then r is:

$$(a) 2$$

$$(b) 3$$

$$(c) 4$$

$$(d) 6$$

[June 2017] [Modified]

Ans. (a) 2

Working Note:

$${}^{16}p_r = 240$$

$$16!/(16-r)! = 240$$

$$16 \times 15 \times 14!/(16-r)! = 240$$

$$14! = (16-r)!$$

$$14 = 16-r$$

$$r = 2$$

Q. 23. KOLKATA is _____ times of LONDON in respect of arrangements of their letters.

$$(a) 5$$

$$(b) 6$$

$$(c) 7$$

$$(d) 8$$

[Dec. 2017] [Modified]

Ans. (c) 7

Working Note:

$$\text{Total Arrangements of KOLKATA} = 7!/2!2!$$

$$= 5040/(2 \times 2)$$

$$= 1260$$

$$\text{Total Arrangements of LONDON} = 6!/2!2!$$

$$= 720/(2 \times 2)$$

$$= 180$$

$$\text{Arrangement of KOLKATA is } 1260/180 = 7 \text{ times of LONDON}$$

Q. 24. The value of 5C_2 :

$$(a) 10$$

$$(b) 9$$

$$(c) 11$$

$$(d) 5$$

[Dec. 2017]

Ans. (a) 10

Working Note:

$5C_2$

$$= 5!/2!3!$$

$$= 120/(2 \times 6)$$

$$= 10$$

Q. 25. ${}^nP_4 = 30 \times {}^nP_2$, then the value of n is:

(a) 10

(b) 8

(c) 6

(d) 5

[June 2018]

Ans. (b) 8

Working Note:

$${}^nP_4 = 30 \times {}^nP_2$$

$$n!/(n-4)! = 30 \times n!/(n-2)!$$

$$(n-2)! = 30 \times (n-4)!$$

$$(n-2)(n-3)(n-4)! = 30(n-4)!$$

$$(n-2)(n-3) = 30$$

$$(n-2)(n-3) = 6 \times 5$$

$$n-2 = 6$$

$$n = 8$$

or

$$n-3 = 5$$

$$n = 8$$

Q. 26. Find the number of ways in which a person can invite his 4 friends selecting at least 1.

(a) 16

(b) 4

(c) 15

(d) 6

[June 2018] [Modified]

Ans. (c) 15

Working Note:

Total number of ways in which one or more can be invited is $= 2^n - 1$

$$= 2^4 - 1$$

$$= 16 - 1$$

$$= 15 \text{ Ways}$$

Q. 27. The value of 5C_2 is equal to:

(a) 5C_1

(b) 5C_3

(c) 5C_4

(d) 5C_5

[June 2018] [Modified]

[June 2019] [Modified]

Ans. (b) 5C_3

Working Note:

We know that, ${}^nC_r = {}^nC_{(n-r)}$

Therefore,

$${}^5C_2 = {}^5C_{(5-2)}$$

$$= {}^5C_3$$

Q. 28. In an examination a candidate has to secure minimum marks in each of the 7 subjects to pass the examination. In how many ways can a student fail?

(a) 128

(b) 127

(c) 126

(d) 125

[Dec. 2018] [Modified]

Ans. (b) 127

Working Note:

A student can fail in one or more subjects.

So total ways in which a student can fail

$$= 2^n - 1$$

$$= 2^7 - 1$$

$$= 128 - 1$$

$$= 127 \text{ Ways}$$

Q. 29. ${}^{2n}P_3 = 84 {}^nP_2$, then the value of n is:

(a) 16

(b) 11

(c) 9

(d) 6

[June 2019]

Ans. (b) 11

Working Note:

$${}^{2n}P_3 = 84 {}^nP_2$$

$$2n!/(2n-3)! = 84 \times n!/(n-2)!$$

$$2n(2n-1)(2n-2) = 84 \times n(n-1)$$

$$4n(2n-1)(n-1) = 84 \times n(n-1)$$

$$(2n - 1) = 21$$

$$2n = 22$$

$$n = 11$$

Q. 30. In how many ways can 8 examination papers be arranged, so that the best and worst papers are never together?

(a) 30,240

(b) 40,320

(c) 5,040

(d) 10,080

Ans. (a) 30,240

Working Note:

Total Arrangements of 8 Examination papers = $8!$ = **40,320**

Total Arrangements with best and worst together = $7! \cdot 2!$ = **10,080**

Total Arrangements with best and worst are never together = $40,320 - 10,080 =$ **30,240 Ways**

Q. 31. The value of ${}^3P_3 - {}^3C_3$ is:

(a) 0

(b) 5

(c) 6

(d) 1

[Dec. 2019]

Ans. (b) 5

Working Note:

$${}^3P_3 - {}^3C_3$$

$$= 6 - 1$$

$$= 5$$

Q. 32. There are 5 questions in group A, 4 in group B and 3 in group C. In how many ways can you select 6 questions taking 3 from group A, 2 from group B and 1 from Group C?

[Dec. 2019] [Modified]

(a) 180

(b) 120

(c) 720

(d) None of these

[Dec. 2019] [Modified]

Ans. (a) 180

Working Note:

Total Ways of selection from Group A = $5C_3 = 10$

Total Ways of selection from Group B = $4C_2 = 6$

Total Ways of selection from Group C = $3C_1 = 3$

Total Ways = $10 \times 6 \times 3 =$ **180 Ways**

Q. 33. In how many ways a committee of 10 persons can be selected from 8 men and 5 women, at least 6 men are there in the committee?

(a) 230

(b) 240

(c) 270

(d) 300

[Dec. 2022]

Ans. (a) 230

Working Note:

Atleast 6 men has to be selected, so either 6 or 7 or 8 men can be selected, Total ways =

$$8C_6 \times 5C_4 + 8C_7 \times 5C_3 + 8C_8 \times 5C_2$$

$$= 28 \times 5 + 8 \times 10 + 1 \times 10$$

$$= 140 + 80 + 10$$

$$=$$
 230 ways

Q. 34. The value of ${}^6C_5 \times {}^6P_6$ is:

(a) 4230

(b) 4340

(c) 4320

(d) 2340

[Dec. 2022]

Ans. (c) 4320

Working Note:

$${}^6C_5 \times {}^6P_6$$

$$= 6 \times 720$$

$$= 4320$$

Q. 35. The number of arrangements of x books in a bookshelf is 5040, then the value x is:

(a) 8

(b) 10

(c) 7

(d) 5

[Dec. 2022]

Ans. (c) 7

Working Note:

Number of arrangements of x books = $x!$, Therefore,

$$x! = 5040$$

$$x! = 7!$$

$$x = 7$$

Q. 36. If ${}^nP_5 = 20 \times {}^nP_3$, find the value of n .

(a) 6

(b) 10

(c) 8

(d) 12

[June 2023]

Ans. (c) 8

Working Note:

$${}^nP_5 = 20 \times {}^nP_3$$

$$n!/(n-5)! = 20 \times n!/(n-3)!$$

$$(n-3)! = 20 \times (n-5)!$$

$$(n-3)(n-4)(n-5)! = 20 \times (n-5)!$$

$$(n-3)(n-4) = 5 \times 4$$

$$n-3 = 5$$

$$n = 8$$

or

$$n-4 = 4$$

$$n = 8$$

Q. 37. ${}^{22}C_r = {}^{22}C_{(2r+1)}$, find the Value of rC_4 ,

(a) 35

(b) 30

(c) 40

(d) 45

[June 2023]

Ans. (a) 35

Working Note:

$${}^{22}C_r = {}^{22}C_{(2r+1)}, \text{ Therefore,}$$

$$r + (2r + 1) = 22$$

$$3r + 1 = 22$$

$$3r = 21$$

$$r = 7$$

$$\text{Now, } {}^rC_4 =$$

$7C_4$

$$= 7!/4!3!$$

$$= 5040/24 \times 6$$

$$= 35$$

Q. 38. There are 7 questions in an examination out of which 2 are difficult. In how many ways can a student select 4 questions amongst them so that 2 difficult questions are excluded?

(a) 10

(b) 5

(c) 35

(d) 8

[June 2023 Dec. 2023]

Ans. (b) 5

Working Note:

4 Questions are to be attempted out of 7, but 2 difficult questions are to be excluded, Hence, 4 questions are to be selected out of 5 questions only.

$$\text{Total ways} = {}^5C_4$$

$$= 5!/4! 1!$$

$$= 120/24$$

$$= 5 \text{ ways}$$

Q. 39. If ${}^{10}P_r$ is 720, then the value of r is:

(a) 3

(b) 4

(c) 5

(d) 6

Ans. (a) 3**Working Note:**

$$10P_r = 720$$

$$10!/(10-r)! = 720$$

$$36,28,800 = 720 (10-r)!$$

$$5040 = (10-r)!$$

$$7! = (10-r)!$$

$$7 = 10-r$$

$$r = 3$$

Q. 40. In how many ways can the colours of the rainbow be arranged so that the red and blue colours are always separated?

(a) 3000

(b) 4000

(c) 4200

(d) 3600

*[Dec. 2023]***Ans. (d) 3600****Working Note:**

$$\text{Total Arrangements} = 7P_7 = 7! = 5040$$

$$\text{Arrangements where red and blue are always together} = 6P_6 \times 2P_2 = 6! \times 2! = 720 \times 2 = 1440$$

$$\text{Arrangements where red and blue are always separated} = 5040 - 1440 = \mathbf{3600} \text{ ways}$$

Q. 41. The LCM of (3!, 4!, 5!) is:

(a) 6

(b) 24

(c) 120

(d) 60

*[June 2024]***Ans. (c) 120****Working Note:**

$$3! = 6$$

$$4! = 24$$

$$5! = 120$$

$$\text{LCM} = 120$$

Q. 42. Express $5 \times 6 \times 7 \times 8 \times 9$ in factorial notation.

(a) $9!/5!$ (b) $9!-5!$ (c) $9!/4!$ (d) $10!-4!$ *[June 2024]***Ans. (c) $9!/4!$** **Working Note:**

$$5 \times 6 \times 7 \times 8 \times 9$$

Multiplying numerator and denominator by $4!$ We get,

$$= 9 \times 8 \times 7 \times 6 \times 5 \times 4!/4!$$

$$= \mathbf{9!/4!}$$

PRACTICE QUESTIONS

Q. 1. In how many ways can 4 people be selected at random from 6 boys and 4 girls if there are exactly 2 girls?

(a) 90

(b) 360

(c) 92

(d) 480

Ans. (a) 90**Working Note:**

$$\text{Number of selections} = {}^6C_2 \times {}^4C_2 = 15 \times 6 = \mathbf{90}$$

Q. 2. The number of parallelograms, formed from a set of six parallel lines intersecting another set of four parallel lines is:

(a) 360

(b) 90

(c) 180

(d) 45

Ans. (b) 90

Working Note:

No. of parallelograms = $6C_2 \times 4C_2 = 15 \times 6 = 90$

Q. 3. There are 5 books on English, 4 Books on Tamil and 3 books on Hindi. In how many ways can these books be placed on a shelf if the books on the same subjects are to be together?

(a) 1,36,800

(b) 1,83,600

(c) 1,03,680

(d) 1,63,800

Ans. (c) 1,03,680

Working Note:

Total 3 types of books will be $3P_3$ and then each book can be arranged internally. Therefore,

No. of arrangements = $5P_5 \times 4P_4 \times 3P_3 \times 3P_3 = 1,03,680$

Q. 4. If six times the number of permutations of 'n' items taken 3 at a time is equal to seven times the number of permutation of (n - 1) items taken 3 at a time, then the value of 'n' will be:

(a) 7

(b) 9

(c) 13

(d) 21

Ans. (d) 21

Working Note:

$$6 \times nP_3 = 7 \times (n-1)P_3$$

$$6 \times n!/(n-3)! = 7 \times (n-1)!/(n-1-3)!$$

$$6 \times n(n-1)!/(n-3)(n-4)! = 7 \times (n-1)!/(n-4)!$$

$$6 \times n/(n-3) = 7$$

$$6n = 7(n-3)$$

$$6n = 7n - 21$$

$$n = 21$$

Q. 5. The number of words that can be formed out of the letters "ARTICLE" so that vowels occupy even place is:

(a) 36

(b) 144

(c) 574

(d) 754

Ans. (b) 144

Working Note:

Even places are 3 and Vowels are A, E, I. Therefore, total arrangements = $3P_3 \times 4P_4 = 6 \times 24 = 144$

Q. 6. The number of ways of arranging 6 boys and 4 girls in a row so that all 4 girls are together is:

(a) 6! 4!

(b) 2 (7! × 4!)

(c) 7! 4!

(d) 2 (6! × 4!)

Ans. (c) 7! 4!

Working Note:

All girls together will form one group, so total arrangements will be $(6+1)!$ And now girls can be arranged internally.

So total Arrangements = 7! 4!

Q. 7. How many numbers not exceeding 1000 can be made from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 if repetition is not allowed.

(a) 364

(b) 585

(c) 728

(d) 81

Ans. (b) 585

Working Note:

Total one digit numbers that can be formed are $9P_1 = 9$

Total two digit numbers that can be formed are $9P_2 = 9 \times 8 = 72$

Total three digit numbers that can be formed are $9P_3 = 9 \times 8 \times 7 = 504$

Four digit numbers with given numbers will always exceed 1000, so not possible.

Total arrangements = $504 + 72 + 9 = 585$

Q. 8. 7 books are to be arranged in such a way so that two particular books are always at first and last place. Find the number of arrangements.

- (a) 60
- (b) 120
- (c) 240
- (d) 480

Ans. (c) 240

Working Note:

Total Arrangements = $2P_2 \times 5P_5 = 2! \times 5! = 2 \times 120 = 240$

Q. 9. Number of ways of painting a face of a cube by 6 colours is:

- (a) 36
- (b) 6
- (c) 24
- (d) 1

Ans. (b) 6

Working Note:

Any of the 6 colours can be used to paint a particular face of cube. So, total possible ways = $6C_1 = 6$

Q. 10. Six persons A, B, C, D, E and F are to be seated at a circular table. In how many ways can this be done, if A must always have either B or C on his right and B must always have either C or D on his right?

- (a) 3
- (b) 6
- (c) 12
- (d) 18

Ans. (d) 18

Working Notes:

Given the restrictions we must have AB or AC and BC or BD. Therefore, we have following alternatives:

- (i) ABC, D, E, F which gives $(4-1)! = 3! = 6$
- (ii) ABD, C, E, F which gives $(4-1)! = 3! = 6$
- (iii) AC, BD, E, F which gives $(4-1)! = 3! = 6$

Total ways = $6 + 6 + 6 = 18$ ways

Q. 11. A building contractor needs three helpers out of ten men supply. In how many ways can these selections take place?

- (a) 36
- (b) 15
- (c) 150
- (d) 120

Ans. (d) 120

Working Notes:

Total possible selections = $10C_3 = \frac{10!}{3!} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$

Q. 12. In how many ways can the letters of the word FAILURE be arranged so that the consonants may occupy only odd positions?

- (a) 576
- (b) 476
- (c) 376
- (d) 276

Ans. (a) 576

Working Note:

Three consonants can occupy any three out of four odd places = $4P_3 = 24$

Remaining 4 vowels can take remaining 4 places = $4P_4 = 24$

Total arrangements = $24 \times 24 = 576$

Q. 13. A Supreme Court Bench consists of 5 judges. In how many ways, the bench can give a majority decision?

(a) 10

(b) 5

(c) 15

(d) 16

Ans. (d) 16

Working Note:

For a majority decision at least 3 out of 5 should vote in favour of a particular decision.

Total ways = $5C_3 + 5C_4 + 5C_5 = 10 + 5 + 1 = 16$

Q. 14. A boy has 3 library tickets and 8 books of his interest in the library. Of these 8, he does not want to borrow Mathematics part-II unless Mathematics part-I is also borrowed? In how many ways can he choose the three books to be borrowed?

(a) 41

(b) 51

(c) 61

(d) 71

Ans. (a) 41

Working Note:

Mathematics Part-II cannot come without Part-I but vice versa is possible. Therefore, possible ways are to either borrow both or borrow only Part-I or none of them.

Total ways = $6C_3 + 2C_2 \times 6C_1 + 1C_1 \times 6C_2 = 20 + 1 \times 6 + 1 \times 15 = 41$

Q. 15. The number of triangles that can be formed by choosing the vertices from a set of 12 points, seven of which lie on the same straight line is:

(a) 185

(b) 175

(c) 115

(d) 105

Ans. (a) 185

Working Notes:

Seven points are on same line so maximum 2 out of those 7 can be taken to form triangle. So, total ways are:

$5C_3 + 7C_1 \times 5C_2 + 7C_2 \times 5C_1$

$= 10 + 7 \times 10 + 21 \times 5$

$= 10 + 70 + 105$

$= 185$